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On the consistency of warm inflation in the presence of viscosity

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Abstract

This paper studies the stability of warm inflationary solutions when the viscous pressure is taken into account. The latter is a very natural and physically motivated ingredient of warm inflation and is seen to widen the stability range of warm inflation. The spectral index parameters, n_s, n_T, and their ratio are derived. The corresponding WMAP7 data are used to fix some parameters of the model. Two specific examples are discussed in detail: (i) a potential given by $V(\phi,T) = v_1(\phi) + v_2(T)$, and (ii) a potential of the form $V(\phi,T) = \alpha\, v_1(\phi)\, v_2(T)$. In both cases, the viscosity has little impact on the said ratio.

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I. INTRODUCTION

Nowadays it is widely admitted that our Universe experienced a very early stage of accelerated expansion (inflation) which served, among other things, to produce the seeds that, in the course of the subsequent eras of radiation and matter dominance, developed into the cosmic structures (galaxies and clusters thereof) that we observe today [1, 2]. Broadly speaking, we might say there are two main competing scenarios of slow roll inflation: cold inflation [3, 4], that ends in a very short and violent phase of reheating, and warm inflation [5, 6], that does not necessitate the latter phase as the inflaton field decays into radiation at a sufficient (damping) rate, say Γ, during the accelerated expansion. This essential feature enables the Universe to smoothly proceed into the radiation phase, indispensable for the big bang nucleosynthesis. A further characteristic of warm inflation is that the radiation temperature satisfies $T > H$. As a consequence, thermal fluctuations result bigger than quantum fluctuations whereby the former likely are the main origin of cosmic structures [7].

Any inflationary model -whether "cold" or "warm"- must fulfill the requirement of stability; that is to say, its inflationary solutions ought to be attractors in the solution space of the relevant cosmological solutions. It means, in practice, that the scalar field, ϕ, must approach an asymptotic attractor characterized by $\dot{\phi} \simeq -(\partial V/\partial\phi)\,(3H)^{-1}$ in cold inflation, and $\dot{\phi} \simeq -(\partial V/\partial\phi)(3H+\Gamma)^{-1}$ in warm inflation (see e.g. [8, 9]). This ensures that the system will stay sufficiently near to the slow-roll solution for many Hubble times. Here V denotes the scalar field potential and H the Hubble expansion rate.

In the case of warm inflation the conditions for stability have been considered by de Oliveira and Ramos [10] and, recently, more fully by Moss and Xiong [11] who allowed the scalar potential and the damping rate to depend not only on the inflaton field but on the temperature of the radiation gas as well. This automatically introduces two further slow-roll parameters and renders the conditions for a successful warm inflationary scenario even less restrictive.

The aim of this paper is to take a further step in the analysis of the stability of warm inflationary solutions by considering the presence of massive particles and fields in the radiation fluid as well as the existence of a the viscous pressure, Π, associated to the resulting mixture of heavy and light particles. This novel quantity is well motivated on physical grounds. It can represent either a genuine viscous pressure (as in radiative fluids [12, 13]), or an effective negative pressure linked to the production of particles by the inflaton field, or the decay of the heavy fields into light fields (see, e.g. [14–17]), or both. Whatever the case, it may significantly influence the dynamics of warm inflation. In particular, the inflationary region occupies a wider section of the phase space

associated to the autonomous system of differential equations when $\Pi \neq 0$ than otherwise [18], and its effect on the matter power spectrum (of a few per cent) will be possibly detected in future experiments [19]. Moreover, the adiabatic index γ of the mixture will not necessary be 4/3 but some value in the range $1 < \gamma < 2$, that will depend on the proportion between heavy and light particles in the mixture.

This paper is organized as follows. Section II presents the basic equations for warm inflation with viscosity. Section III analyzes the stability for inflationary solutions. Section IV determines the power spectrum produced by the thermal fluctuations. Corrections to the spectral index are considered when the viscosity comes into play. Section V applies the full theory to two specific examples differing in the type of potential but with identical forms for the viscous pressure and damping rate, Γ. The seven year WMAP data are used to constrain the model. Finally, Section VI summarizes and discusses our findings. Hence forward a coma means partial derivative. We use units in which $c = \hbar = 8\pi G = 1$.

II. BASIC EQUATIONS

We consider a spatially flat, homogeneous and isotropic universe dominated by an scalar field ϕ and radiation, of temperature T, endowed with a viscous pressure $\Pi = -3\,\zeta H$, where ζ is the semi-positive definite coefficient of bulk viscosity. A potential $V(\phi, T)$ and the damping rate $\Gamma(\phi, T)$ dictate the evolution of the field so that the corresponding Klein-Gordon equation assumes the form

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V_{,\phi} = 0\,. \tag{1}$$

In this scenario, the Friedmann equation reads

$$3H^2 = \rho_\phi \, + \, \rho_\gamma\,, \tag{2}$$

where $\rho_\phi = (1/2)\dot{\phi}^2 \, + \, V(\phi, T)$ and $\rho_\gamma = Ts$, denote the energy densities of the scalar field (the inflaton) and radiation, respectively. Here

$$s \simeq -V_{,T}\,, \tag{3}$$

is the entropy density. This expression follows from the thermodynamical relation $s = -\partial f/\partial T$, when the Helmholtz free-energy, $f = (1/2)\dot{\phi}^2 \, + \, V(\phi, T) \, + \, \rho_\gamma \, - \, Ts$, is dominated by the potential.

Since for an expanding Universe the viscous pressure is necessarily negative the total pressure

$$p = \frac{1}{2}\dot{\phi}^2 \, - \, V(\phi, T) \, + \, (\gamma - 1)Ts \, + \, \Pi, \tag{4}$$

is lower than in the absence of viscosity. Obviously, the immediate effect of Π is to further accelerate the expansion. Clearly, the adiabatic coefficient, γ, must slowly vary with expansion but, for simplicity sake, we will consider it constant.

The conservation of the stress-energy can be expressed as

$$T\dot{s} + 3H(\gamma T s + \Pi) = \Gamma\dot{\phi}^2 \,. \tag{5}$$

Making $u = \dot{\phi}$ and introducing the dimensionless strength dissipation parameter, $Q = \Gamma/(3H)$, the slow roll equations take the form

$$u = \frac{-V_{,\phi}}{3H(1+Q)}, \qquad Ts = \frac{Qu^2 + 3H\zeta}{\gamma}, \qquad 3H^2 = V(\phi, T) \,. \tag{6}$$

The number of e-folds is given by

$$N = \int_{\phi_*}^{\phi_{end}} \frac{3H^2(1 + Q)}{V_{,\phi}(\phi, T)}\, d\phi \,, \tag{7}$$

where ϕ_* and ϕ_{end} denote the value of the inflaton field when the perturbations cross the horizon (i.e., when $k = aH$) and at the end of inflation, respectively.

Finally, in warm inflation the customary slow roll parameters,

$$\epsilon = \frac{1}{2}\left(\frac{V_{,\phi}}{V}\right)^2, \quad \eta = \frac{V_{,\phi\phi}}{V}, \qquad \beta = \frac{V_{,\phi}\Gamma_{,\phi}}{V\Gamma}\,, \tag{8}$$

are supplemented by two others,

$$b = \frac{TV_{,\phi T}}{V_{,\phi}}\,, \qquad c = \frac{T\Gamma_{,T}}{\Gamma}\,, \tag{9}$$

that gauge the temperature dependence of the potential and the damping rate, respectively.

III. STABILITY ANALYSIS

We are interested in finding the impact of the viscous pressure, Π, on the stability of the slow roll. This immediately implies that we must focus on the strong regime (i.e., $Q \gg 1$) as in the weak regime ($Q \ll 1$) viscosity will be practically non-existent [19].

To find the conditions for the validity of the slow roll approximation, we perform a linear stability analysis to see whether the system remains close to the slow roll solution for many Hubble times. In cold inflationary scenario, the slow roll equation is of first order in the time derivative. Choosing the inflaton field as independent variable, the conservation equations (1) and (5) can be written as first order equations in the derivative with respect to ϕ, indicated by a prime,

$$x' = F(x)\,, \tag{10}$$

where

$$x = \begin{pmatrix} u \\ s \end{pmatrix} . \tag{11}$$

Thus, the system (1), (5) becomes

$$u' = -3H \; - \Gamma - \; V_{,\phi} u^{-1} \, , \tag{12}$$

$$s' = -3H\gamma s u^{-1} \; - \; 3H\Pi (Tu)^{-1} \; + T^{-1}\Gamma u \, . \tag{13}$$

Here the Hubble rate and temperature are determined by (2) and (3), respectively.

Taking a background $\overline{x}$ which satisfies the slow roll equations (6), the linearized perturbations satisfy

$$\delta x' = M(\overline{x}) \delta x - \overline{x}' \, , \tag{14}$$

where

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} , \tag{15}$$

is the matrix of first derivatives of F evaluated at the slow roll solution. Linear stability demands that its determinant be positive and its trace negative.

The matrix elements read,

$$A = \frac{H}{u} \left\{ -3(1+Q) \; - \; \frac{\epsilon}{(1+Q)^2} \right\} , \tag{16}$$

$$B = \frac{H}{s} \left\{ -c\,Q - \frac{Q}{(1+Q)^2} \epsilon \; + \; b(1+Q) \right\} , \tag{17}$$

$$C = \gamma \frac{Hs}{u^2} \left(6 - \frac{\epsilon}{(1+Q)^2} \right) \left\{ 1 + \frac{\Pi}{\gamma^2 \rho_\gamma} \left(\frac{6(1+Q)^2 - 2\epsilon}{6(1+Q)^2 - \epsilon} \right) \right\} , \tag{18}$$

$$D = \gamma \frac{H}{u} \left(c - 4 - \frac{Q\epsilon}{\gamma^2 (1+Q)^2} \right) \; + \; \frac{H\Pi}{u\gamma\rho_\gamma} \left\{ c - \frac{Q\epsilon}{\gamma^2 (1 \, + \, Q)^2} \; + \; \frac{3\Pi}{2\gamma^2 \rho_\gamma} \right\} . \tag{19}$$

In the strong regime ($Q \gg 1$), the determinant and trace of M assume the comparatively simple expressions

$$\det M = \frac{3\gamma Q H^2}{u^2} \left(4 - 2b + c + (c-2b) \frac{\Pi}{\gamma^2 \rho_\gamma} \; - \; \frac{3}{2} \frac{\Pi^2}{\gamma^4 \rho_\gamma^2} \right) , \tag{20}$$

and

$$\mathrm{tr}M = \frac{H}{u}\left\{-3Q + \gamma(c-4) + \frac{\Pi}{2\gamma^3\rho_\gamma}\left(2\gamma^2 c + 3\frac{\Pi}{\rho_\gamma}\right)\right\}. \tag{21}$$

Sufficient conditions for stability are that M varies slowly and that

$$|c| \leq \frac{4 - 3\sigma^2/2}{1+\sigma} - 2b\,, \qquad b \geq 0\,, \tag{22}$$

where $\sigma \equiv \frac{\Pi}{\gamma^2\rho_\gamma}$. Upon these conditions the determinant results positive and the trace negative, implying stability of the corresponding solution. Expression (22.1) generalizes Eq. (27) of Moss and Xiong [11].

Since the chosen background is not an exact solution of the complete set of equations, the forcing term in equation (14) depends on $\overline{x}'$, and will be valid only if $\overline{x}'$ is small. The size of $\overline{x}'$ depends on the quantities $\dot{u}/(Hu)$ and $\dot{s}/(Hs)$. From the time derivative of (6.3) we obtain

$$\frac{\dot{H}}{H^2} = -\frac{\epsilon}{1+Q}\,. \tag{23}$$

Combining this with the other slow-roll equations, (6.1) and (6.2), we get

$$\frac{\dot{u}}{Hu} = \frac{1}{\Delta}\left[-\frac{c[A(1+Q) - BQ] - 4}{1+Q}\epsilon + \frac{4Q}{1+Q}\beta + (Ac-4)\eta - \frac{3(1+Q)c}{1-f}b\right], \tag{24}$$

and

$$\frac{\dot{s}}{Hs} = \frac{3}{\Delta}\left[\frac{A(3+Q) - B(1+Q)}{1+Q}\epsilon + \frac{Q-1}{1+Q}A\beta - 2A\eta - \frac{(1+Q)[Ac(Q-1)+Q+1]c}{(1-f)Q}b\right], \tag{25}$$

where

$$\Delta = 4(1+Q) + Ac(Q-1)\,, \qquad A = \frac{\rho_\gamma + \gamma^{-1}\Pi}{\rho_\gamma - \kappa\Pi}\,, \qquad B = \frac{\Pi}{\rho_\gamma - \kappa\Pi}\,, \tag{26}$$

$$f = -\frac{3}{2}\frac{(1+Q)^2}{Q}\frac{\zeta}{\gamma H\epsilon}\,, \qquad \kappa = \rho_\gamma\frac{\zeta_{,\rho_\gamma}}{\zeta}\,. \tag{27}$$

Notice that when $\Pi \to 0$, one has that $A \to 1$, $B \to 0$, $f \to 0$, and therefore the equations (23)-(25) reduce to the corresponding expressions in [11]. Obviously, the value of the parameter κ in this limit depends on the specific expression of the viscosity coefficient, ζ; but it does not alter the value of B in the said limit. In this limit, the κ parameter could take any value depending of the model. Its value does not affect the $\Pi \to 0$ limit.

IV. DENSITY FLUCTUATIONS

The thermal fluctuations produce a power spectrum of scalar density fluctuations of the form [11]

$$\mathcal{P}_s = \frac{\sqrt{\pi}}{2}\frac{H^3 T}{u^2}\sqrt{1+Q}\,. \tag{28}$$

Note that the power spectrum of fluctuations in inflationary models where the friction coefficient depends also on the temperature, i.e., $\Gamma = \Gamma(\phi, T)$, was considered recently in Ref.[20].

We calculate the spectral index by means of

$$n_s - 1 = \frac{\dot{\mathcal{P}}_s}{H\mathcal{P}_s} \,. \tag{29}$$

By virtue of the equations (23)-(25), we obtain

$$n_s - 1 = \frac{p_1\epsilon + p_2\beta + p_3\eta + p_4 b}{\Delta} \,, \tag{30}$$

where the p_i coefficients are given by

$$p_1 = -\frac{10(2\,+\,Q) - A(\,3\,+5\,c\,+\,Q)\,+B\,(1\,+\,Q\,+\,(5c/2)Q)}{1\,+\,Q} \,, \tag{31}$$

$$p_2 = \frac{A(Q\,-\,1)\,-\,10Q}{1\,+\,Q}, \tag{32}$$

$$p_3 = \frac{8(1\,+\,Q) - A(2\,+\,2c\,+2Q\,+\,3cQ)}{1\,+\,Q}, \tag{33}$$

$$p_4 = \frac{3(1\,+\,Q)[1\,+\,(1+5c/2)Q]}{(1\,-\,f)Q} \,. \tag{34}$$

For $Q \gg 1$, and assuming c of order unity, the p_i coefficients reduce to

$$p_1 = -10 + A - B(1+5c/2)\,, \quad p_2 = A - 10, \quad p_3 = 8 - A(2+3c)\,, \quad p_4 = \frac{3Q(1+5c/2)}{(1-f)}\,, \tag{35}$$

and $\Delta = Q(4 + Ac)$. Therefore (30) becomes

$$n_s - 1 = -\frac{10 - A + B(1+5c/2)}{(4+Ac)Q}\,\epsilon\, - \,\frac{10-A}{(4+Ac)Q}\,\beta + \frac{8 - A(2+3c)}{(4+Ac)Q}\,\eta\, + \frac{3(1+5c/2)}{(4+Ac)(1-f)}b\,. \tag{36}$$

The tensor modes happen to be the same as in the cold inflationary models [11], i.e.,

$$\mathcal{P}_T = H^2 \,, \tag{37}$$

and the corresponding spectral index is

$$n_T\,-\,1 = -\frac{2}{1+Q}\epsilon \,. \tag{38}$$

With the help of of (37), (28) and (6.1) the tensor-to-scalar amplitude ratio can be written as

$$r = \frac{2\,V_{,\phi}(\phi, T)}{9\sqrt{\pi}\,H^3\,T(1\,+\,Q)^{5/2}} \,. \tag{39}$$

The recent WMAP seven-year results imply the upper-bound $r < 0.36$ (95% CL) [21] on the scalar-tensor ratio. Below, shall make use of this bound to set constraints on the parameters of our models.

V. EXAMPLES

In this section we apply the formalism of above to two specific cases sharing the same damping and bulk viscosity coefficient but differing in the potential. In general, the damping coefficient may be written as

$$\Gamma(\phi, T) = \Gamma_0 \left(\frac{\phi}{\phi_0}\right)^m \left(\frac{T}{\tau_0}\right)^n, \tag{40}$$

with n and m real numbers and ϕ_0, τ_0, and Γ_0 some nonnegative constants. The damping term has a generic form given approximately by $\Gamma \sim g^4\phi^2\tau$, where g is the coupling constant [23]. From Ref.[24] the damping term, $\tau = \tau(\phi, T)$, is related to the relaxation time of the radiation and for the models with an intermediate particle decay, $\tau = \tau(\phi)$ is linked to the lifetime of the intermediate particle. Different choices of n and m have been adopted. For instance the case $n = m = 0$ was considered by Taylor and Berera [7], whereas the choice $m = 2$, $n = -1$ corresponds to the damping term first calculated by Hosoya [24]. This expression slightly differs from those in [23, 25], where a single index rather than two was considered.

As for the bulk viscosity coefficient we use the general expression

$$\zeta = \zeta_0\, \rho_\gamma^\lambda\,, \tag{41}$$

where ζ_0 is a positive semi-definite constant and λ an integer that may take any of the two values: $\lambda = 1/2$, i.e., $\zeta \propto \rho_\gamma^{1/2}$[26] (see also Ref.[27]) and $\lambda = 1$, i.e., $\zeta \propto \rho_\gamma$ [19].

A. First example: a separable potential

We begin by considering the comparatively easy separable potential [11]

$$V(\phi, T) = v_1(T) + v_2(\phi)\,, \tag{42}$$

where $v_1(T) = -\frac{\pi^2}{90} g_* T^4 - \frac{1}{12} m_\phi^2 T^2$ and $v_2(\phi) = \frac{1}{2} m_\phi^2 \phi^2$.

In this case, the strength of dissipation parameter and the slow rolls parameters are

$$Q = \frac{\Gamma(\phi, T)}{\sqrt{3(v_1 + v_2)}}\,, \tag{43}$$

and

$$\epsilon = \frac{m_\phi^4 \phi^2}{2V^2}\,, \qquad \beta = \frac{m\, m_\phi^2}{V}\,, \qquad \eta = \frac{m_\phi^2}{V}\,, \qquad b = 0\,, \qquad c = n \tag{44}$$

respectively.

Recalling that $\rho_\gamma = -T\,V_{,T}$, we obtain the following expression for the radiation energy density

$$\rho_\gamma = \frac{2\pi^2}{45} g_* T^4 + \frac{1}{6} m_\phi^2 T^2 \,. \tag{45}$$

To get a lower limit to the friction term in the presence of viscosity, we follow the method of [11]. We cast Eq. (28) in the form

$$\mathcal{P}_s \approx \frac{T^4}{u^2} \frac{H^3}{T^3} (1+Q)^{1/2} \,, \tag{46}$$

and use (6.2) together with (40) to arrive to the cubic equation

$$4X^3 - g_2 X - g_3 = 0 \,, \tag{47}$$

with

$$g_2 = -\frac{12\zeta\mathcal{P}_s}{Q^* T^3} \,, \qquad g_3 = \frac{8\pi^2\gamma}{45} \frac{g_* \mathcal{P}_s}{Q^*} \,, \tag{48}$$

where $Q^* \equiv Q\,\sqrt{1+Q}$ and $X \equiv H/T$.

The only real solution of the Eq. (47) is found to be

$$X = Z\,\sinh(\theta) \,, \tag{49}$$

with

$$Z = 2\sqrt{\frac{\zeta\mathcal{P}_s}{Q^* T^3}} \,, \quad \text{and} \quad \theta = \frac{1}{3}\sinh^{-1}\left(\frac{\pi^2\gamma}{45}\sqrt{\frac{g_*^2 Q^* T^9}{\zeta^3 \mathcal{P}_s}}\right) \,. \tag{50}$$

Now, the condition for warm inflation, $T > H$ (i.e., $X < 1$), translates into

$$Q > g_* \mathcal{P}_s \left[1 - \frac{3}{2}\left(\frac{45}{\pi^2\gamma g_*}\right)\frac{\zeta}{T^3}\right] \,. \tag{51}$$

Note that, as $\zeta \geq 0$, the viscous pressure augments the range of Q, and that the condition for warm inflation without viscosity, $Q > g_*\,\mathcal{P}_s$, is trivially recovered in the limit $\zeta \to 0$.

Using (41) and (45) the inequality (51) can be written as

$$Q > g_* \mathcal{P}_s \left[1 - \frac{3\zeta_0}{\gamma}\left(\frac{2\pi^2 g_*}{45}\right)^{\lambda-1} T^{4\lambda-3}\right] \,. \tag{52}$$

Since $Q > 0$ an upper bound on ζ_0 follows, namely,

$$\zeta_0 < \zeta_{max}^{(1)} \equiv \frac{\gamma}{3}\left(\frac{2\pi^2 g_*}{45}\right)^{1-\lambda} T^{3-4\lambda} \,. \tag{53}$$

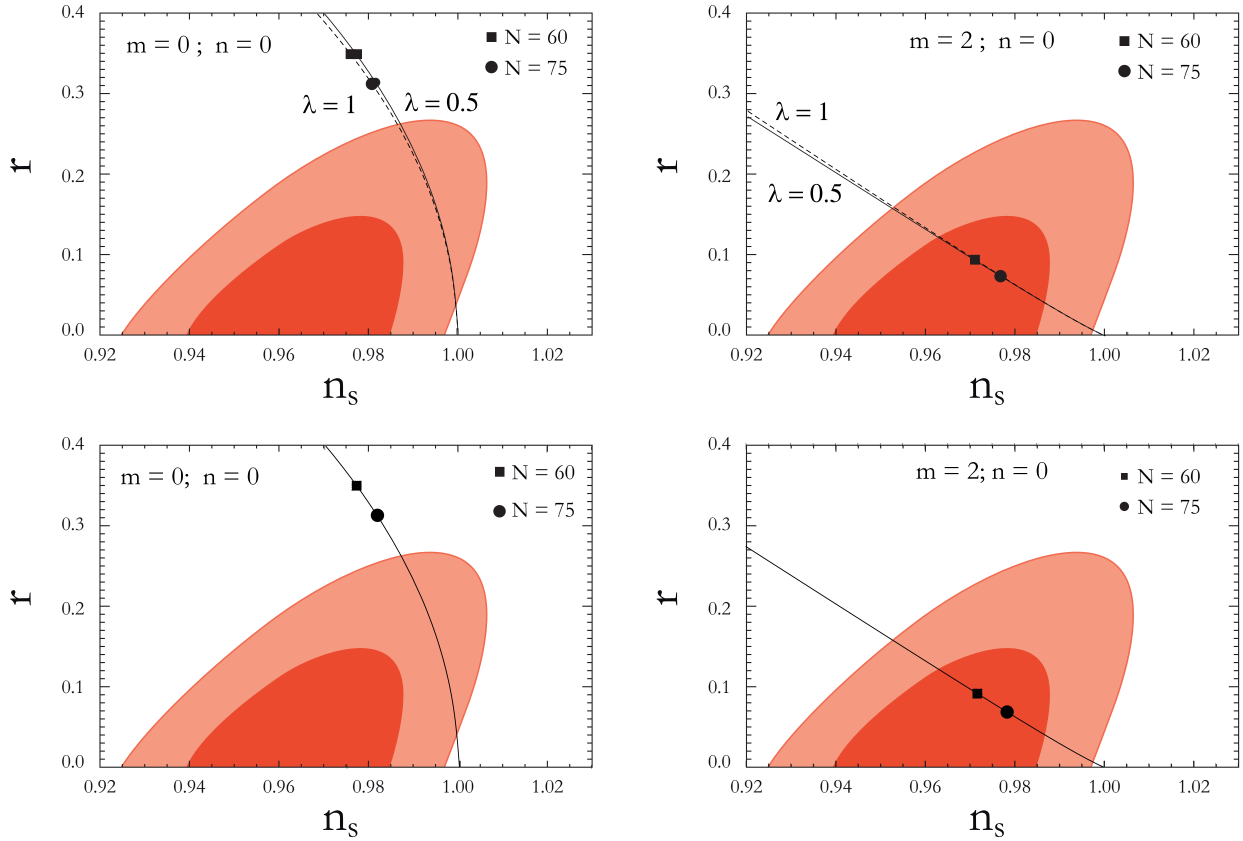

FIG. 1: Top row of panels: Plot of the tensor-scalar ratio r as a function of the spectral index n_s, for two values of the λ parameter in the case of example 1 (i.e., potential (42)). Bottom row: Same as the top row but assuming no viscosity ($\zeta_0 = 0$). In each panel the 68% and 95% confidence levels set by seven-year WMAP experiment are shown. The latter places severe limits on the tensor-scalar ratio [21].

Figure 1 depicts the dependence of the tensor-scalar ratio, r, on the spectral index, n_s, for the model given by Eqs. (40), (41), and (42) when $\lambda = 0.5$ and when $\lambda = 1$. From Ref. [21], two-dimensional marginalized constraints (68% and 95% confidence levels) on inflationary parameters r and n_s, the spectral index of fluctuations, defined at $k_0 = 0.002$ Mpc^{-1}. The seven-year WMAP data [21] places stronger bounds on r than the five-year WMAP data [22]. In order to write down values that relate n_s and r, we used Eqs. (36) and (39), and the values $g_* = 100$, $\gamma = 1.5$, $\zeta_0 = (2/3)\zeta^{(1)}_{max}$, and $m_\phi = 0.75 \times 10^{-5}$, $T = 2.5 \times 10^{-6}$, $\Gamma_0 = 1.2 \times 10^{-6}$, $\tau_0 = 3.73 \times 10^{-5}$, $\phi_0 = 0.3$ for $m = 0, n = 0$; and $m_\phi = 2.5 \times 10^{-5}$, $T = 1.75 \times 10^{-6}$, $\Gamma_0 = 3.58 \times 10^{-6}$, $\tau_0 = 5.63 \times 10^{-5}$, $\phi_0 = 0.6$ for $m = 2, n = 0$, in Planck units [23].

Figure1 suggests that the pair of indices ($m = 2$, $n = 0$), corresponding to the right panel, is preferred over the other pair of indices ($m = n = 0$), left panel. Likewise, it shows that there is little difference between choosing $\lambda = 1$ or $\lambda = 0.5$ as well as with the case of no viscosity, i.e., $\zeta_0 = 0$.

Panel in Fig. 1	N	r	N	r
top left ($m = n = 0$)	60	0.351	75	0.314
top right ($m = 2$, $n = 0$)	60	0.094	75	0.074

TABLE I: Results from first example with $\lambda = 1$ (The results for $\lambda = 1/2$ are very similar). Rows from top to bottom refers to panels of Fig. 1 from left to right.

Panel in Fig. 1	N	r	N	r
bottom left ($m = n = 0$)	60	0.350	75	0.318
bottom right ($m = 2$, $n = 0$)	60	0.094	75	0.074

TABLE II: Results from first example with no viscosity, i.e., $\zeta_0 = 0$.

Table II indicates the value of the ratio r for $\lambda = 1$ and different choices of the pair of indices m and n when the number of e-folds is 60 and when it is 75. Very similar values (not shown) follow for $\lambda = 0.5$. All of them can be checked with the help of Eqs. (36) and (39).

A comparison of the results shown in both Tables indicates that only in the case of the pair $(m = n = 0)$ with $N = 75$ (top and bottom left panels in Fig. 1) viscosity makes a non-negligible impact.

B. Second example: effective Coleman-Weinberg potential

As a second example we consider the effective Coleman Weinberg potential near the critical point, for $T < T_c$ in the region about ϕ_B [6]

$$V(\phi, T) = \alpha\, v_1(T)\, v_2(\phi)\,, \tag{54}$$

where $v_1 = k - T^2 \log(\delta/T)$ with $\delta \sim m_{\rm Pl}$, and $v_2 = (\phi - \phi_B)^2$. To ensure that $V(\phi, T) \geq 0$ the condition $k \geq \frac{\delta^2}{2e} = k_{min}$ must be imposed.

The strength parameter adopts the form

$$Q = \frac{\Gamma(\phi, T)}{\sqrt{3\alpha\, v_1\, v_2}}\,, \tag{55}$$

whereas the slow-rolls parameters are given by

$$\epsilon = \eta = \frac{2}{(\phi - \phi_B)^2}\,, \qquad \beta = \frac{2m}{\phi(\phi - \phi_B)}\,, \qquad b = \frac{T^2[1 - 2\log(\delta/T)]}{v_1}\,, \qquad c = n\,. \tag{56}$$

Panel in Fig. 2	N	r	N	r
top left ($m = n = 0$)	60	0.177	75	0.158
top right ($m = 2,\ n = 0$)	60	0.225	75	0.193

TABLE III: Results from the second example with $\lambda = 1$ (very similar results, not shown, follow for $\lambda = 1/2$). Rows from top to bottom refers to panels of Fig. 2 from left to right.

To enforce the smallness of the three first slow-rolls parameters, the inflaton field must fulfill $\phi \gg \phi_B + \sqrt{2}$.

Thus, the energy density of the radiation fluid can be written as

$$\rho_\gamma = \alpha\,(\phi - \phi_B)^2\, T^2\,(2\log(\delta/T) - 1)\,. \tag{57}$$

Following the procedure of the previous example, we find a lower bound for Q, namely,

$$Q > \frac{\alpha \mathcal{P}_s}{4}\left(\gamma(\phi - \phi_B)^2 \frac{(2\log[\delta/T) - 1]}{T^2} - \frac{3}{\alpha}\frac{\zeta}{T^3}\right)\,, \tag{58}$$

and using (57), this relation simplifies to

$$Q > \frac{\gamma \rho_\gamma \mathcal{P}_s}{4T^4}\left(1 - \frac{3}{\gamma}\,\zeta_0\, T\, \rho_\gamma^{\lambda-1}\right)\,. \tag{59}$$

Clearly, in this case as well, viscosity increases the range of Q.

The corresponding upper bound on ζ_0, is

$$\zeta_0 < \zeta_{max}^{(2)} \equiv \frac{\gamma \rho_\gamma^{1-\lambda}}{3T}\,. \tag{60}$$

Figure 2 illustrates the dependence of the tensor-scalar ratio on the spectral index, from Eqs. (36), (39), and (54). We have used the following values: $\alpha = 3 \times 10^{-10}$, $\gamma = 1.5$, $\zeta_0 = (2/3)\zeta_{max}^{(2)}$, $\phi_B = 2$, $\delta = 1$, $k = (3/2)\,k_{min}$ and $\alpha = 10^{-11}$, $T = 2.5 \times 10^{-6}$, $\Gamma_0 = 1.2 \times 10^{-6}$, $\tau_0 = 3.73 \times 10^{-5}$, $\phi_0 = 0.3$ for $m = 0, n = 0$; and $\alpha = 10^{-8}$, $T = 1.75 \times 10^{-6}$, $\Gamma_0 = 3.58 \times 10^{-6}$, $\tau_0 = 5.63 \times 10^{-5}$, $\phi_0 = 0.6$ for $m = 2, n = 0$, in Planck units [23].

Figure 2 hints that none of the two pair of indices is clearly preferred over the other. On the other hand, it shows that there is practically no difference between the choices $\lambda = 1$ and $\lambda = 1/2$.

Table III indicates the value of the ratio r for $\lambda = 1$ and different choices of the pair of indices m and n when the number of e-folds is 60 and when it is 75. Very similar values (not shown) follow for $\lambda = 0.5$. All of them can be checked with the help of Eqs. (36) and (39).

Again, a the results of both Tables show, in this example, the impact of viscosity is rather low.

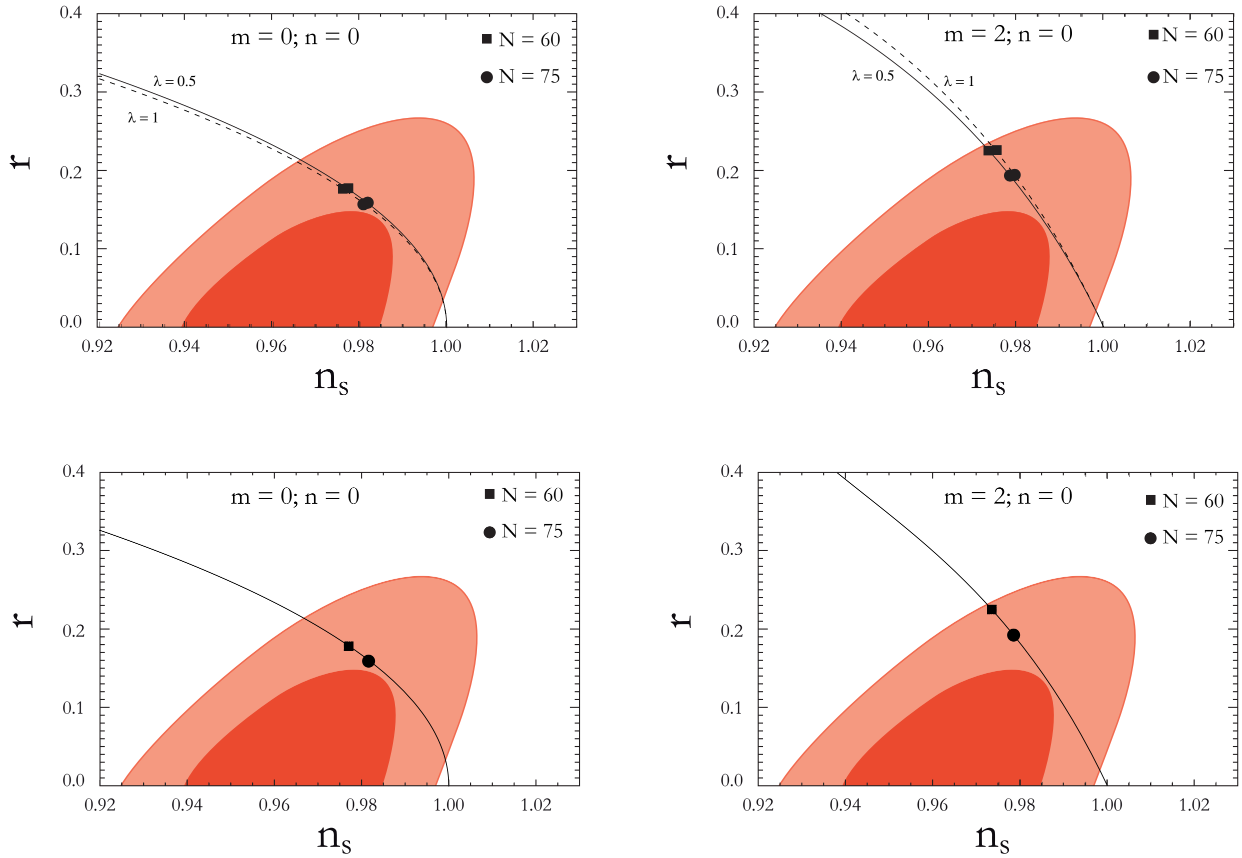

FIG. 2: Top row of panels: Plot of the tensor-scalar ratio r as a function of the spectral index n_s, for two values of the λ parameter in the case of example 2 (i.e., potential (54)). Bottom row: Same as the top row but assuming no viscosity ($\zeta_0 = 0$). In each panel the 68% and 95% confidence levels set by seven-year WMAP experiment are shown. The latter places severe limits on the tensor-scalar ratio [21].

Panel in Fig. 2	N	r	N	r
bottom left ($m = n = 0$)	60	0.175	75	0.159
bottom right ($m = 2$, $n = 0$)	60	0.224	75	0.193

TABLE IV: Results from the second example in the absence of viscosity, i.e., $\zeta_0 = 0$.

VI. CONCLUSIONS

We studied various aspects of warm inflationary universe models when viscosity is taken into account. The latter (whether physical or effective) is a very general feature in multiparticle and entropy producing systems and, in the context of warm inflation, it is of special significance when the rate of particle production and/or interaction is high. In consequence we have focused on the strong regime $Q \gg 1$ (in the weak regime viscosity is, at best, negligible [19]).

Equation (22) gives the necessary and sufficient condition for the existence of stable slow-roll solutions. Under this condition, we got the same stability range obtained in the no-viscous case,

so long as $\sigma = -8/3$. In this sense, the range of the slow-roll parameter c decreases when $-8/3 < \sigma < 0$, and increases when $\sigma < -8/3$.

We calculated a general expression for the spectral index, n_s (Eq. (30)), that depends explicitly on viscosity through the four p_i coefficients. The latter do not depend on the slow-roll parameters $(\epsilon, \beta, \eta, \text{and}\, b)$, as shown by equations (31)-(34).

In order to further ensure the stability of the warm viscous inflation, the slow-roll parameters must satisfy the following conditions

$$\epsilon \ll 1 + Q\,, \quad |\beta| \ll 1 + Q\,, \quad |\eta| \ll 1 + Q\,,$$

as well as the condition on the slow-roll parameter that describes the temperature dependence of the potential, namely,

$$|b| \ll \frac{(1-f)Q}{1+Q}\,.$$

To get explicit expressions we have considered two examples for the scalar potential. In the first case, $V(\phi, T) = v_1(\phi) + v_2(T)$ (Eq. (42)), it trivially follows that b vanishes; on the other hand, the stability condition requires that the strength parameter fulfils

$$Q > g_* \mathcal{P}_s \left(1 - \frac{\zeta_0}{\zeta_{max}^{(1)}}\right)\,.$$

Hence, the viscous pressure increases the stability range of Q as $\zeta_0 < \zeta_{max}^{(1)}$, where $\zeta_{max}^{(1)}$ is given by Eq. (53).

The second example is provided by the potential $V(\phi, T) = \alpha v_1(\phi) v_2(T)$, (Eq. (54)). In this case, the stability condition for Q is

$$Q > \frac{\gamma \rho_\gamma \mathcal{P}_s}{4T^4} \left(1 - \frac{\zeta_0}{\zeta_{max}^{(2)}}\right)\,,$$

with $\zeta_{max}^{(2)}$ given by (60). Again, the range of stability of Q augments in the presence of viscosity.

To bring in some explicit results we have taken the constraint $n_s - r$ plane to first-order in the slow roll approximation. For the first potential (Eq. (42)) we obtained that, when $\lambda = 0.5$ and $\lambda = 1$, the model is consistent with the WMAP seven year data for the pair of indices ($m = 2$, $n = 0$), see Fig. 1. Likewise, for the second potential (Eq.(54)) all two choices of the (m, n) pair show compatibility with the said data -see Fig.1. The results obtained indicate that the effect of viscosity on the value of r is marginal. However, it cannot be discarded that future experiments uncover it.

We should note that there are some properties in this model that deserve further study. In particular, we did not address the case $n \neq 0$. In fact, when $\Gamma = \Gamma(\phi, T)$ a more detailed and

laborious calculation for the density perturbation would be necessary in order to check the validity of expression (28). We intend to return to this point in the near future by following an approach analogous to that of Ref.[20].

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[1] A.R. Liddle and D.H Lyth, *Cosmological Inflation and Large-scale Structure* (Cambridge University Press, Cambridge, 2000).

[2] D.H Lyth and A.R. Liddle, *The Primordial Density Perturbation* (Cambridge University Press, Cambridge, 2009).

[3] A. Linde, Phys. Lett. B **108**, 389 (1982).

[4] A. Linde, *Particle Physics and Inflationary Cosmology* (Harwood Academic, Chur, Switzerland, 1990).

[5] A. Berera and L.Z. Fang, Phys. Rev. Lett. **74**, 1912 (1995).

[6] A. Berera, Phys. Rev. Lett. **75**, 3218 (1995).

[7] A. Taylor and A. Berera, Phys. Rev. D **62**, 083517 (2000).

[8] D.S. Salopek and J.R. Bond, Phys. Rev. D **42**, 3936 (1990).

[9] A.R. Liddle, P. Parsons and J.D. Barrow, Phys. Rev. D **50**, 7222 (1994).

[10] H.P. de Oliveira and R.O. Ramos, Phys. Rev. D **57**, 741 (1998).

[11] I.G. Moss and C. Xiong, JCAP 0811(2008)023.

[12] S. Weinberg, Astrophys. J. **168**, 175 (1971).

[13] S. Weinberg, *Gravitation and Cosmology* (J. Wiley, New York, 1972).

[14] Ya.B. Zeldovich, JETP Lett. **12**, 307 1970.

[15] B.L. Hu, Phys. Lett. A **90**, 375 (1982).

[16] J.D. Barrow, Nucl. Phys. B **310**, 743 (1988).

[17] W. Zimdahl and D. Pavón, Phys. Lett. A **176**, 57 (1993).

[18] J.P. Mimoso, A. Nunes, and D. Pavón, Phys. Rev. D **73**, 023502 (2006).

[19] S. del Campo, R. Herrera, and D. Pavón, Phys. Rev. D **75**, 083518 (2007).

[20] C. Graham and I. G. Moss, JCAP **0907**, 013 (2009).

[21] D. Larson *et al.*, arXiv:1001.4635 [astro-ph.CO].

[22] J. Dunkley *et al.*, Astrophys. J. Suppl. **180**, 306 (2009); G. Hinshaw *et al.*, Astrophys. J. Suppl. **180**, 225 (2009).

[23] L. M. H. Hall, I. G. Moss, and A. Berera, Phys. Rev. D **69**, 083525 (2004).

[24] A. Hosoya and M.-aki Sakagami, Phys. Rev. D 29, 2228 (1984).

[25] Y. Zhang, JCAP 0903(2009)023.

[26] W. J. Li, Y. Ling, J. P. Wu and X. M. Kuang, Phys. Lett. B **687**, 1 (2010)

[27] I. Brevik and O. Gorbunova, Gen. Rel. Grav. **37**, 2039 (2005); I. Brevik, Int. J. Mod. Phys. D **15**, 767 (2006); X. M. Kuang and Y. Ling, JCAP 10(2009)024.